Sample Paper -1
SUMMATIVE ASSESSMENT –I
Class – X Mathematics

Time allowed: 3 hours	**ANSWERS**	Maximum Marks: 90

SECTION – A

1. We have

$$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots\ldots \cos 180^\circ$$

LHS $= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 89^\circ \cos 90^\circ \cos 91^\circ \ldots \cos 180^\circ$ = R.H.S

$$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots 0 \times \cos 90^\circ \cos 91^\circ \ldots \cos 180^\circ = 0$$

2. Product of two numbers = Product of their LCM and HCF

$$\Rightarrow 1800 = 12 \times LCM \Rightarrow LCM = \frac{1800}{12} = 150.$$

3. Since the given lines are parallel

$$\therefore \frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1} \; i.e., k = \frac{15}{4}$$

4. Let us first form the frequency table for the given data as given below:

Value (x_i)	110	120	130	140
Frequency (f_i)	2	4	2	2

We observe that the value 120 has the maximum frequency.
Thus, the mode is 120.

SECTION – B

5. ΔABC is right-angled at C.

$\therefore AB^2 = AC^2 + BC^2$ [By Pythagoras theorem]

$\Rightarrow AB^2 = AC^2 + AC^2$ [$\because SC = BC$]

$\Rightarrow AB^2 = 2AC^2$

6. Let f(x) = $x^2 + 2x + 5$

$$= x^2 + 2x + 1 + 4$$

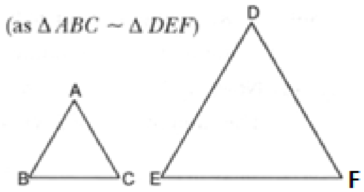
$$= (x+1)^2 + 4$$

Now, for every real value of x, $(x+1)^2 \geq 0$

$\Rightarrow$ For every real value of x, $(x+1)^2 + 4 \geq 4$

$\therefore$ For every real value of x, $f(x) \geq 4$ and hence it has no zero.

7. We have, $\frac{area\,of\ \Delta ABC}{area\,of\ \Delta DEF} = \frac{BC^2}{EF^2}$



$\Rightarrow \frac{64}{121} = \frac{BC^2}{EF^2} \Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2}$

$\Rightarrow \frac{BC}{15.4} = \frac{8}{11}$

$\therefore\ BC = \frac{8}{11} \times 15.4 = 11.2cm$

8. If x is irrational, then $y = \frac{x}{2}$ is also an irrational number such that $0 < y < x$.

If x is rational, then $\frac{x}{\sqrt{2}}$ is an irrational number such that $\frac{x}{\sqrt{2}} < x$ as $\sqrt{2} > 1$.

$\therefore$ $y = \frac{x}{\sqrt{2}}$ is an irrational number such that $0 < y < x$.

9. $3\sin^2 20^o - 2\tan^2 45^o + 3\sin^2 70^o$

$= 3\sin^2 20^o(90^o - 70^o) - 2(1)^2 + 3\sin^2 70^o$ $[\because \tan 45^o = 1]$

$= 3\cos^2 70^o - 2 + 3\sin^2 70^o$ $[\because \sin(90-\theta) = \cos\theta]$

$= 3(\sin^2 70^o + \cos^2 70^o) - 2$

$= 3 \times 1 - 2 = 3 - 2 = 1.$ $[\because \sin^2\theta + \cos^2\theta = 1]$

10. Since α and β are the zeros of the polynomial $f(x) = x^2 - px + q$,

$\therefore$ $\alpha + \beta = -\left(\frac{-p}{1}\right) = p$ and $\alpha\beta = \frac{q}{1} = q$

Thus, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{p}{q}$

SECTION – C

11. We know that an old positive integer n is of the form (4q+1) or (4q+3) for some integer q.

Case I When n=(4q+1)

In this case

$n^2 - 1 = (4q+1)^2 - 1 = 16q^2 + 8q = 8q(2q+1)$

Which is clearly divisible by 8.

Case II When n=(4q+3)

In this case, We have

$n^2 - 1 = (4q+3)^2 - 1 = 16q^2 + 14q + 8 = 8(2q^2 + 3q + 1)$

Which is clearly divisible by 8.

12. Let $a - d$, a and$a + d$ be the zeros of the polynomial $f(x)$. Then,

Sum of the zeros = $\frac{Coefficient\ of\ x^2}{Coefficient\ of\ x^3}$

$\Rightarrow \quad (a-d)+a+(a+d) = -\frac{(-p)}{1}$

$\Rightarrow \quad 3a = p \quad \Rightarrow \quad a = \frac{p}{3}$

Since a is a zero of the polynomial $f(x)$. Therefore,

$f(a) = 0$

$\Rightarrow \quad a^3 - pa^2 + qa - r = 0$

$\Rightarrow \quad \left(\frac{p}{3}\right)^3 - p\left(\frac{p}{3}\right)^2 + q\left(\frac{p}{3}\right) - r = 0 \qquad \left[\because a = \frac{p}{3}\right]$

$\Rightarrow \quad p^3 - 3p^3 + 9pq - 27r = 0$

$\Rightarrow \quad 2p^3 - 9pq + 27r = 0$

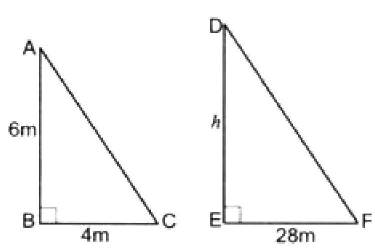
13. Let AB be a vertical pole of length 6 m and BC be its shadow and DE be tower and EF b its shadow. Join AC and DF.

Now, in $\Delta ABC\ and\ \Delta\, DEF$, we have

$\angle B = \angle C = 90^o$ (Angle of elevation of the sun)

$\angle C = \angle F$

$\therefore \Delta ABC \sim \Delta DEF$ (By AA criterion of similarity)



Thus, $\frac{AB}{DE} = \frac{BC}{EF}$

$\Rightarrow \frac{6}{h} = \frac{4}{28}$ (Let DE=h)

$\Rightarrow \frac{6}{h} = \frac{1}{7} \Rightarrow h = 42$

Hence, height of tower, DE = 42 m

14. We know that the sum of the opposite angles of a cyclic quadrilateral is 180°. In the cyclic quadrilateral ABCD, angles A and C and angles B and D form pairs of opposite angles.

$\therefore$ $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

$\Rightarrow$ $2x - 1 + 2y + 15 = 180°$ and $y + 5 + 4x - 7 = 180°$

$\Rightarrow$ $2x + 2y = 166°$ and $4x + y = 182°$

$\Rightarrow$ $x + y = 83°$ ...(i)

And, $4x + y = 182°$ ...(ii)

Subtracting equation (i) from equation (ii), we get

$3x = 99 \Rightarrow x = 33$

Substituting $x = 33$ in equation (i), we get $y = 50$

Hence, $\angle A = (2x-1)° = (2\times 33 - 1)° = 65°$, $\angle B = (y+5)° = (50+5)° = 55°$

$\angle C = (2y+15)° = (2\times 50 + 15)° = 115°$ and $\angle D = (4x-7)° = (4\times 33 - 7)° = 125°$

15.

Calculation of median

Class intervals	**Frequency (*f*)**	**Cumulative frequency (*cf*)**
0-100	2	2
100-200	5	7
200-300	x	$7 + x$
300-400	12	$19 + x$
400-500	17	$36 + x$
500-600	20	$56 + x$
600-700	y	$56 + x + y$
700-800	9	$65 + x + y$
800-900	7	$72 + x + y$

900-1000	4	$76 + x + y$
		Total = 100

We have, $N = \sum fi = 100$

$\Rightarrow \quad 76 + x + y = 100$

$\Rightarrow \quad x + y = 24$

It is given that the median is 525. Clearly, it lies in the class 500-600.

$\therefore \quad l = 500, h = 100, f = 20, F = 36 + x$ and $N = 100$

Now, median = $l + \dfrac{\frac{N}{2} - F}{f} \times h$

$$\Rightarrow \quad 525 = 500 + \frac{\frac{100}{2} - (36 + x)}{20} \times 100$$

$$\Rightarrow \quad 525 - 500 = \frac{50 - 36 - x}{20} \times 100$$

$\Rightarrow \quad 25 = (14 - x) \times 5$

$\Rightarrow \quad 25 = 70 - 5x$

$\Rightarrow \quad 5x = 45$

$\Rightarrow \quad x = \dfrac{45}{5} \Rightarrow \quad x = 9$

Putting $x = 9$ in $x + y = 24$, we get

$9 + y = 24$

$\Rightarrow \quad y = 24 - 9 = 15$

Thus, $x = 9$ and $y = 15$.

16. We have AB = 4 and BC = 3

By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC = \sqrt{AB^2 + BC^2}$

$\Rightarrow \quad AC = \sqrt{4^2 + 3^2}$

$\Rightarrow \quad AC = \sqrt{25} = 5$

When we consider the t-ratios of $\angle A$, we have

Base = AB = 4, Perpendicular = BC = 3 and Hypotenuse = AC = 5

$\therefore$ $\sin A = \frac{BC}{AC} = \frac{3}{5}$, $\cos A = \frac{AB}{AC} = \frac{4}{5}$, $\tan A = \frac{BC}{AB} = \frac{3}{4}$

$\text{cosec} A = \frac{AC}{BC} = \frac{5}{3}$, $\sec A = \frac{AC}{AB} = \frac{5}{4}$ and $\cot A = \frac{AB}{BC} = \frac{4}{3}$

17. Let AB = BC = x.

It is given that ΔABC is a right-angled at B.

$\therefore$ $AC^2 = AB^2 + BC^2$

$\Rightarrow$ $AC^2 = x^2 + x^2$

$\Rightarrow$ $AC = \sqrt{2}x$

It is given that

$\Delta ABE \sim \Delta ACD$

$$\Rightarrow \frac{\text{Area}(\Delta ABE)}{\text{Area}(\Delta ACD)} = \frac{AB^2}{AC^2}$$

$$= \frac{x^2}{\left(\sqrt{2}x\right)^2}$$

$$= \frac{1}{2}$$

18. Suppose my age is x years and my son's age is y years. Then,

$x = 3y$ ...(i)

5 years later, my age will be $(x + 5)$ years and my son's age will be $(y + 5)$ years.

$\therefore$ $x + 5 = \frac{5}{2}(y + 5)$

$\Rightarrow$ $2x - 5y - 15 = 0$ ...(ii)

Putting $x = 3y$ in equation (ii), we get

$6y - 5y - 15 = 0$ $\Rightarrow$ $y = 15$

Putting $y = 15$ in equation (i), we get

$x = 45$

19. L.H.S$= \frac{1-\sin\theta}{1+\sin\theta}$

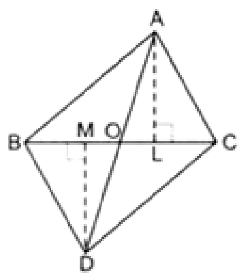
$= \frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$ [Rationalising the denominator]

$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta} = \left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$
$$= (\sec\theta - \tan\theta)^2 = RHS$$

20. Given: ΔABC *and* ΔDBC are the on the same base BC and AD intersects BC at O.

To Prove: $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Construction: Draw AL $\perp$ BC and DM $\perp$ BC



Proof: In ΔAOL *and* ΔDMO, w have

$\angle ALO = \angle DMO = 90^o$ *and*

$\angle AOL = \angle DOM$ (Vertically opposite angles)

$\therefore \Delta ALO \sim \Delta DMO$ (By AA-Similarity)

$\Rightarrow \frac{AL}{DM} = \frac{AO}{DO}$ ...(*i*)

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} BC \times AL}{\frac{1}{2} BC \times DM} = \frac{AL}{DM} = \frac{AO}{DO}$ (Using (i))

Hence, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

SECTION – D

21. Let the numerator and denominator of the fraction be *x* and *y* respectively.

Then,

Fraction = $\frac{x}{y}$

It is given that

Denominator = 2(Numerator) + 4

$\Rightarrow \quad y = 2x + 4$

$\Rightarrow \quad 2x - y + 4 = 0$

According to the given condition, we have

$y - 6 = 12(x - 6)$

$\Rightarrow \quad y - 6 = 12x - 72$

$\Rightarrow \quad 12x - y - 66 = 0$

Thus, we have the following system of equations

$2x - y + 4 = 0$ ...(i)

$12x - y - 66 = 0$ ...(ii)

Subtracting equation (i) from equation (ii), we get

$10x - 70 = 0$

$\Rightarrow \quad x = 7$

Putting $x = 7$ in equation (i), we get

$14 - y + 4 = 0$

$\Rightarrow \quad y = 18$

Hence, required fraction = $\frac{7}{18}$

22. LH.S = $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)$

$= \frac{\sec^2 A}{\cos ec^2 A}$

$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$

R.H.S. = $\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$

$= \left(\frac{1-\tan A}{\frac{\tan A - 1}{\tan A}}\right)^2 = \left(\frac{1-\tan A}{\tan A - 1} \times \tan A\right)^2$

$(-\tan A)^2 = \tan^2 A$

L.H.S = R.H.S

23. We have,

$$x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$\Rightarrow \quad x\sin\theta(\sin^2\theta) + y\cos\theta(\cos^2\theta) = \sin\theta\cos\theta$

$\Rightarrow \quad x\sin\theta(\sin^2\theta) + x\sin\theta(\cos^2\theta) = \sin\theta\cos\theta \qquad [\because x\sin\theta = y\cos\theta]$

$\Rightarrow \quad x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$

$\Rightarrow \quad x\sin\theta = \sin\theta\cos\theta$

$\Rightarrow \quad x = \dfrac{\sin\theta\cos\theta}{\sin\theta} = \cos\theta$

Now,

$$x\sin\theta = y\cos\theta$$

$\Rightarrow \quad \cos\theta\sin\theta = y\cos\theta \qquad [\because x = \cos\theta]$

$\Rightarrow \quad y = \dfrac{\cos\theta\sin\theta}{\cos\theta} = \sin\theta$

$\therefore \quad x^2 + y^2 = \sin^2\theta + \cos^2\theta = 1$

24. Here, we have the cumulative frequency distribution. So, first we convert it into an ordinary frequency distribution. We observe that there are 80 students getting marks greater than or equal to 0 and 77 students have secured 10 and more marks. Therefore, the number of students getting marks between 0 and 10 is 80 – 77 = 3.

Similarly, the number of students getting marks between 10 and 20 is 77 – 72 = 5 and so on.

Thus, we obtain the following frequency distribution:

Marks	**Number of students**	**Marks**	**Number of students**
0-10	3	50-60	15
10-20	5	60-70	12
20-30	7	70-80	6
30-40	10	80-90	2
40-50	12	90-100	8

Now, we compute arithmetic mean by taking 55 as the assumed mean.

Computation of mean

Marks (x_i)	Mid-value	Frequency(f_i)	$u_i = \frac{x_i - 55}{10}$	$f_i u_i$
0-10	5	3	–5	–15
10-20	15	5	–4	–20
20-30	25	7	–3	–21
30-40	35	10	–2	–20
40-50	45	12	–1	–12
50-60	55	15	0	0
60-70	65	12	1	12
70-80	75	6	2	12
80-90	85	2	3	6
90-100	95	8	4	32
Total		$\sum f_i = 80$		$\sum f_i u_i = -26$

We have,

$N = \sum f_i = 80, \sum f_i u_i = -26$, A = 55 and h = 10

$\therefore \quad \overline{X} = A + h\left\{\frac{1}{N}\sum f_i u_i\right\}$

$= 55 + 10 \times \frac{-26}{80} = 55 - 3.25 = 51.75$ marks

25. By division algorithm, we have

$f(x) = g(x) \times q(x) + r(x)$

$\Rightarrow \quad f(x) - r(x) = g(x) \times q(x)$

$\Rightarrow \quad f(x) + \{-r(x)\} = g(x) \times q(x)$

Clearly, RHS is divisible by $g(x)$. Therefore, LHS is also divisible by $g(x)$. Thus, if we add $-r(x)$ to $f(x)$, then the resulting polynomial is divisible by $g(x)$. Let us now find the remainder when $f(x)$ is divided by $g(x)$.

$$
\begin{array}{r|l}
 & 4x^2-6x+22 \\
x^2+2x-3 & 4x^4+2x^3-2x^2+x-1 \\
 & 4x^4+8x^3-12x^2 \\
 & -\quad -\quad + \\
\hline
 & -6x^3+10x^2+x-1 \\
 & -6x^3-12x^2+18x \\
 & +\quad +\quad - \\
\hline
 & 22x^2-17x-2 \\
 & 22x^2+44x-66 \\
 & -\quad -\quad + \\
\hline
 & -61x+65
\end{array}
$$

$\therefore \quad r(x) = -61x + 65$

Thus, we should add $-r(x) = 61x - 65$ to $f(x)$ so that the resulting polynomial is divisible by $g(x)$.

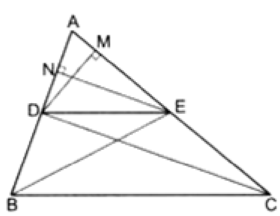
26. Given: A triangle ABC in which a line Parallel to sides BC intersect other two sides AB and AC at D and E respectively.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$.

Construction: Join BE and CD and then draw DM $\perp$ AC an EN $\perp$ AB.

Proof: Area of $\Delta ADE = \left(\frac{1}{2} base \times height\right)$

So, $ar(\Delta ADE) = \frac{1}{2} AD \times EN$



And $ar(\Delta BDE) = \frac{1}{2} DB \times EN$

Similarly, $ar(\Delta ADE) = \frac{1}{2} AE \times DM$

And $ar(\Delta DEG) = \frac{1}{2} EC \times DM$

Therefore, $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} AD \times EN}{\frac{1}{2} DB \times EN} = \frac{AD}{DB}$ ...(i)

And $\frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{\frac{1}{2} AE \times DM}{\frac{1}{2} EC \times DM} = \frac{AE}{EC}$ ...(ii)

Now, $\Delta BDE \text{ and } \Delta DEG$ are on the same base DE and between the same parallel lines BC and DE.

So, $ar(\Delta BDE) = ar(\Delta DEG)$ ...(iii)

Therefore, from (i), (ii) and (iii) we have,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Second Part

As DE||BC

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad \Rightarrow \frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

$$\Rightarrow \frac{AD+DB}{DB} + \frac{AE+EC}{EC} \Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$$

$$\Rightarrow AB = AC \quad (As\, DB = EC)$$

$\therefore \Delta ABC$ is an isosceles triangle.

27. Graph of the equation $2x + y = 2$:

When $y = 0$, we have $x = 1$

When $x = 0$, we have $y = 2$

Thus, we obtain the following table giving coordinates of two points on the line represented by the equation $2x + y = 2$.

x	1	0
y	0	2

Graph of the equation $2x + y = 6$:

When $y = 0$, we get $x = 3$

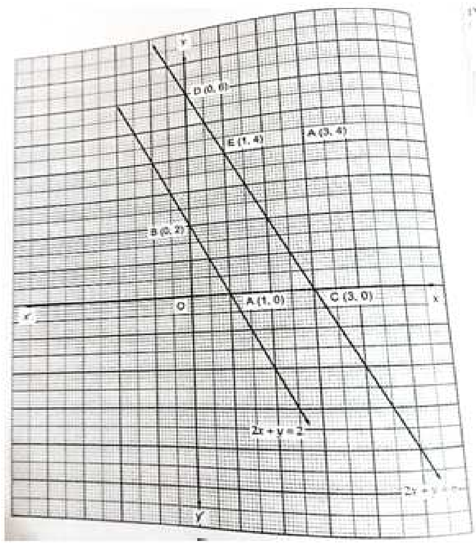
When $x = 0$, we get $y = 6$

Thus, we obtain the following table giving coordinates of two points on the line represented by the equation $2x + y = 6$.

x	3	0
y	0	6

Plotting points A(1, 0) and B (0, 2) on the graph paper on a suitable scale and drawing a line passing through them, we obtain the graph of the line represented by the equation $2x + y = 2$ as shown in the graph.

Plotting points C(3, 0) and D(0, 6) on the same graph paper and drawing a line passing through them, we obtain the graph of the line represented by the equation $2x + y = 6$ as shown in the graph.



Clearly, lines AB and CD form trapezium ACDB.

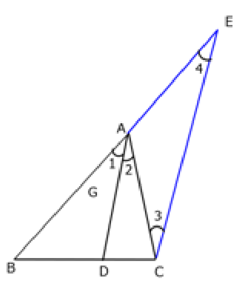
Also, area of trapezium ACDB = Area of ΔOCD – Area of ΔOAB

$$= \frac{1}{2}(OC \times OD) - \frac{1}{2}(OA \times OB)$$

$$= \frac{1}{2}(3 \times 6) - \frac{1}{2}(1 \times 2) = 8 \text{ sq.units}$$

28. Given: AΔABC in which AD is the internal bisector of ∠A and meets BC in D.

To prove: $\frac{BD}{DC} = \frac{AB}{AC}$



Construction: Draw CE || DA to meet BA produced in E.

Proof: Since CE || DA and AC cuts them,

$\therefore \quad \angle 2 = \angle 3$ [Alternate angles] ...(i)

And, $\angle 1 = \angle 4$ [Corresponding angles] ...(ii)

But, $\angle 1 = \angle 2$ [$\because$ AD is the bisector of $\angle A$]

From (i) and (ii), we get

$\angle 3 = \angle 4$

Thus, in ΔACE, we have

$\angle 3 = \angle 4$

$\Rightarrow \quad AE = AC$ [Sides opposite to equal angles are equal] ...(iii)

Now, in ΔBCE, we have

DA || CE

$\Rightarrow \quad \frac{BD}{DC} = \frac{BA}{AE}$ [Using Basic Proportionality Theorem]

$\Rightarrow \quad \frac{BD}{DC} = \frac{AB}{AC}$ [$\because$ BA = AB and AE = AC (From (iii)]

Thus, $\frac{BD}{DC} = \frac{AB}{AC}$

29. To represent the data in the table graphically, we mark the upper limits of the class interval on x-axis and their corresponding cumulative frequency on y-axis choosing a convenient scale.

Now, let us plot the points corresponding to the ordered pair given by (38, 0), (40, 3), (42, 5), (44, 9), (46, 14), (48, 28), (50, 32) and (52, 35) on a graph paper and join them by a freehand smooth curve.

Thus, the curve obtained is the less than type ogive.

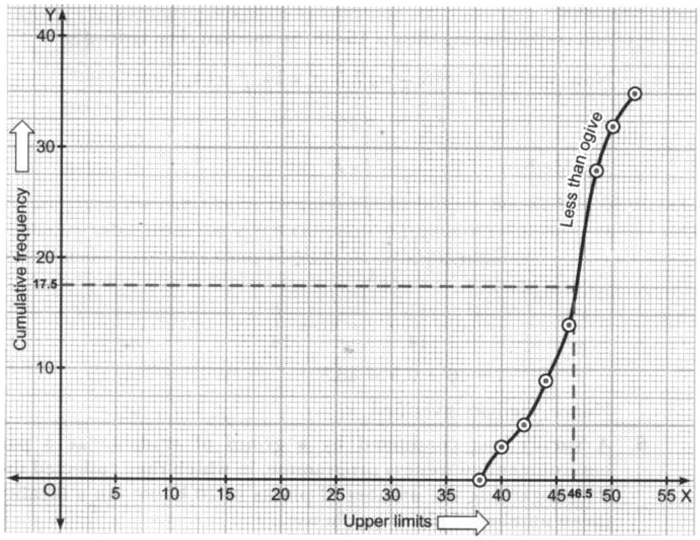


Fig. 6.3

Now, locate $\frac{n}{2} = \frac{35}{2} = 17.5$ on the y-axis,

We draw a line from this point parallel to x-axis cutting the curve at a point from this point, draw a perpendicular line to the x-axis. The point of intersection of this perpendicular with the x-axis gives the median of **the** data. Here it is 46.5

Let us make the following table in order to find median by using formula.

Weight (in kg)	No. of Students frequency (f_i)	Cumulative frequency (cf)
36-38	0	0
38-40	3	6
40-42	2	5
42-44	4	9
44-46	5	14
46-48	14	28
48-50	4	32
50-52	3	35

Total	$\sum f_i = 35$	

Here, $n = 35, \frac{n}{2} = \frac{35}{2} = 17.5$, cumulative frequency greater than $\frac{n}{2} = 17.5$ is 28 and corresponding class is 46-48. So median class is 46 - 48.

Now, we have $l = 46, \frac{n}{2} = 17.5, cf = 14, f = 14, h = 2$

$$\therefore Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$= 46 + \left(\frac{17.5 - 14}{14} \right) \times 2$$

$$= 46 + \frac{3.5}{14} \times 2 = 46 + \frac{7}{14}$$

$$= 46 + 0.5 = 46.5$$

Hence, median is verified.

30. We have,

LHS $= \frac{(1 + \cot A + \tan A)(\sin A - \cos A)}{\sec^3 A - \operatorname{cosec}^3 A}$

$$= \frac{\left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)(\sin A - \cos A)}{\left(\frac{1}{\cos^3 A} - \frac{1}{\sin^3 A}\right)}$$

$$= \frac{\left(1 + \frac{\cos^2 A + \sin^2 A}{\sin A \cos A}\right)(\sin A - \cos A)}{\left(\frac{\sin^3 A - \cos^3 A}{\sin^3 A \cos^3 A}\right)}$$

$$= \frac{\sin A \cos A + \cos^2 A + \sin^2 A}{\sin A \cos A} \times \frac{\sin^3 A \cos^3 A}{\sin^3 A - \cos^3 A}(\sin A - \cos A)$$

$$= \frac{\sin A \cos A + 1}{\sin^3 A - \cos^3 A} \times (\sin^2 A \cos^2 A)(\sin A - \cos A)$$

$$= \frac{(\sin A \cos A + 1)(\sin^2 A \cos^2 A)(\sin A - \cos A)}{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)} \qquad \left[\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)\right]$$

$= \dfrac{(\sin A \cos A + 1)\sin^2 A \cos^2 A}{1 + \sin A \cos A}$

$= \sin^2 A \cos^2 A$ = RHS

31. (i) Let the number of child patients in the hospital be x.

Then, the number of male patients = 3x

And, the number of female patients = 2(3x) = 6x

According to the question,

6x + 3x + x = 900

$\Rightarrow$ 10x = 900

$\Rightarrow$ x = $\dfrac{900}{10}$ =90

Thus, the number of child patients in the hospital is 90.

And, the number of male patients = 3 × 90 = 270

The number of female patients = 2 × 270 = 540

(i) The values depicted by Rohan's father in the question are charity and empathy.